BICIMAL NUMBERS

By Centre for Elites & Decroly Education Centre

The bicimals can be referred to as the rational binary numbers. A bicimals is the base-two analogue of a decimal; it has a bicimal point and bicimal places and can be terminating or repeating.

Like decimals, bicimals are created from fractions through long division. Also like decimals, bicimals can be converted *back* to fractions. You convert a bicimal to a fraction the same way you convert a decimal to a fraction, you just work in binary instead of decimal and use powers of two instead of powers of ten.

Converting a bicimal fraction into a Decimal Fraction

The process of converting a binary fraction into its decimal equivalent is done in two steps with the numbers the left-hand and right-hand sides of the radix point separately.

When looking at converting the binary on the left-hand side of the radix point we convert it just as we would when converting any binary integer number into its decimal equivalent.

Example 2.13:

$$101_2 = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$= (1 \times 4) + (0 \times 2) + (1 \times 1) = 5$$

Next, we work out the chunk at the right-hand side of the radix point. We do exactly the same thing here as we did on the left, just with the fractional columns:

$$0.101_2 = (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$$

$$= (1 \times \frac{1}{2}) + (0 \times \frac{1}{4}) + (1 \times \frac{1}{8})$$

$$= (1 \times 0.5) + (0 \times 0.25) + (1 \times 0.125) = 0.625_{10}$$

$$\therefore 0.101_2 = 0.625_{10}$$

So we have a fractional part that represents 0.625_{10}.

So we now combine the integer part (5) and fractional parts (0.625) together on either side of the radix point. This gives us the number 5.625_{10}.

A terminating bicimal is always easy to convert to a fraction: the numerator of the resulting fraction is the bicimal itself, treated as an integer; the denominator is 2^n, where *n* is the number of bicimal places.

Example 2.12: Convert 0.1101 to a fraction and then to base ten

Solution

$$0.1101 = \frac{1101}{2^4} = \frac{1101}{10000}$$

This is in decimal equal to $\frac{13}{16} = 0.8125$

Example 2.13: Convert the number 1011.0011_2 to base ten.

Solution:

$$(1011.0011)_2 = \left((1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0) + (0\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4})\right)_{10}$$
$$= 11.1875$$

Converting from a Decimal Fraction to a Binary Fraction (Bicimal)

This is also done in two steps with the two sides of the decimal number separately.

Example 2. 14: Convert 9.12510 to bicimal

Solution

We shall have to deal with the left-hand side of the radix point first. 9 will be converted as seen above

$$9_{10} = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 1001_2$$

Next we need to deal with the fractional part of our decimal number (which as a reminder is 0.125). Again, there is a simple step-by-step method for performing the conversion.

To begin with we take the decimal fraction and multiply it by two (i.e. $2_{10} \times 0.125_{10} = 0.250_{10}$). We then take the whole number part of the result as the first binary digit after the radix point. In this case it is 0 so we have got as far as $0.125_{10} = 0.0?_2$

Next we disregard the whole number part of the previous result (i.e. ignore the 0 before the radix point) and multiply the result by two again. The whole number part of this new result is the second digit after the radix point (i.e. $2_{10} \times 0.250_{10} = 0.50_{10}$). In this case, the whole number part is again a 0 so we have now got as far as $0.125_{10} = 0.00?_2$

Again, disregarding the whole number part of the result and again multiply by 2 (i.e. $2_{10} \times 0.50_{10} = 1.0_{10}$). Again we take the whole number part, this time as the value of the third digit after the radix point. In this case the whole number part is a 1 so we have now got to $0.125_{10} = 0.001?_2$

Again we drop the whole number part but as the fractional part we have left is 0. As we have nothing left we are done. This leaves us with our final representation; 0.125_{10} is exactly equivalent to 0.001_2.

Now that we have extracted both the integer part of our original number and the fractional part we can finally combine them on either side of the radix point:

$$9.125_{10} = 1001.001_2$$

Example 2.15: Convert the decimal number 11.1875_{10} to base 2

Solution

First, look at the integer part: 11.

Table 2.1:

	Quotient	Remainder
11/2	5	$1 = a_0$
5/2	2	$1 = a_1$
2/2	1	$0 = a_2$
1/2	0	$1 = a_3$

Hence

$$\begin{aligned}(11)_{10} &= (a_3 a_2 a_1 a_0)_2 \\ &= (1011)_2\end{aligned}$$

Now let us look at the decimal part, that is, 0.1875.

Table 2.2: Converting a base-10 fraction to binary representation.

	Number	Number after decimal	Number before decimal
0.1875×2	0.375	0.375	$0 = a_{-1}$
0.375×2	0.75	0.75	$0 = a_{-2}$
0.75×2	1.5	0.5	$1 = a_{-3}$
0.5×2	1.0	0.0	$1 = a_{-4}$

Hence

$$\begin{aligned}(0.1875)_{10} &= (a_{-1} a_{-2} a_{-3} a_{-4})_2 \\ &= (0.0011)_2\end{aligned}$$

Having calculated $11_{10} = 1011_2$

and $0.18755_{10} = 0.0011_2$

we have$11.1875_{10} = 1011.1011_2$

Infinite Fractions

Whilst binary and decimal fractions both work on the same principles, each has their own problems when it comes to representing numbers accurately with a given number of digits.

In both cases there are certain numbers that will always result in something called a rounding error, where the number cannot be represented exactly and the nearest number has to be used instead.

Example 2.16: Find the binary equivalent of 0.3 is summarized in Table 3.

Table 3. Converting a base-10 fraction to approximate binary representation.

	Number	Number after decimal	Number before decimal
0.3×2	0.6	0.6	$0 = a_{-1}$
0.6×2	1.2	0.2	$1 = a_{-2}$
0.2×2	0.4	0.4	$0 = a_{-3}$
0.4×2	0.8	0.8	$0 = a_{-4}$
0.8×2	1.6	0.6	$1 = a_{-5}$

As you can see the process will never end. In this case, the number can only be approximated in binary format, that is,

$$(0.3)_{10} \approx (a_{-1}a_{-2}a_{-3}a_{-4}a_{-5})_2 = (0.01001)_2$$

Q: But what is the mathematics behinds this process of converting a decimal number to binary format?

A: Let z be the decimal number written as

$$z = x.y$$

where

x is the integer part and y is the fractional part.

We want to find the binary equivalent of x. So we can write

$$x = a_n 2^n + a_{n-1} 2^{n-1} + \ldots + a_0 2^0$$

If we can now find $a_0, \ldots, a_n$ in the above equation then

$$(x)_{10} = (a_n a_{n-1} \ldots a_0)_2$$

We now want to find the binary equivalent of y. So we can write

$$y = b_{-1} 2^{-1} + b_{-2} 2^{-2} + \ldots + b_{-m} 2^{-m}$$

If we can now find $b_{-1}, \ldots, b_{-m}$ in the above equation then

$$(y)_{10} = (b_{-1} b_{-2} \ldots b_{-m})_2$$

2.1 ADDING AND SUBTRACTING BINARY NUMBERS

It is possible to add and subtract binary numbers in a similar way to base 10 numbers.

For example, 1 + 1 + 1 = 3 in base 10 becomes 1 + 1 + 1 = 11 in binary.

In the same way, 3 – 1 = 2 in base 10 becomes 11 – 1 = 10 in binary.

When you add and subtract binary numbers you will need to be careful when ***'carrying'*** or ***'borrowing'*** as these will take place more often.

1 + 0 = 1
1 + 1 = 10
1 + 1 + 1 = 11
1 – 0 = 1
10 – 1 = 1
11 – 1 = 10

Example 2.17: Calculate, using binary numbers:

(a) 111 + 100 = **1011**

(b) 101 + 110 = **1011**

(c) 1111 + 111 = **10110**

Example 2.18: Calculate the binary numbers:

(a) 111 – 101 = **10**

(b) 110 – 11 = **11**

(c) 1100 – 101 = **111**

SELF-ASSESSMENT ACTIVITY

1. Calculate the binary numbers:

(a)	11 + 1
(b)	11 + 11
(c)	111 + 11
(d)	111 + 10
(e)	1110 + 111
(f)	1100 + 110
(g)	1111 + 10101
(h)	1100 + 11001
(i)	1011 + 1101
(j)	1110 + 10111
(k)	1110 + 1111
(l)	11111 + 11101

2.

3. Calculate the binary numbers:

(a) 11 – 10

(b) 110 – 10

(c) 1111 – 110

(d) 100 – 10

(e) 100 – 11

(h) 11011 – 110

(i) 1111 – 111

(j) 110101 – 1010

(k) 11011 – 111

4. Solve the following equations, where all numbers, including *x*, are binary:

(a) $x + 11 = 1101$

(b) $x - 10 = 101$

(c) $x - 1101 = 11011$

(d) $x + 1110 = 10001$

(e) $x + 111 = 11110$

(f) $x - 1001 = 11101$